Students' Actuarial Society

Education Committee

Actuarial and Financial Mathematics B- Revision Session

This course is challenging but it is possible to score well in the exam.

In this session I will try and give you more examples of some of the more basic questions that you may face in an exam. Knowing how to do these will not guarantee you will do well in the exam, but should put you in a better position to pass the course.

It is important that you understand the concepts in order to be able to apply them to scenarios and questions you will have never seen before. The concepts in this course form the basis for courses in 3rd and 4th year so it is crucial you fully understand all aspects of this course and not just memorise formulas.

This session is here to hopefully reinforce the learning of some of the basics, to hopefully put everyone in the best possible position to get a pass in the course.

Section 1

This section targets the first two main areas of the course which involves obtaining real interest rates using the RPI/CPI given and extending this knowledge to solving problems dealing with index-linked gilts. Stochastic interest rate models are covered in this section as well.

Examples

1) On 15 August 2005 a financial institution issued a bond with term to redemption of 15 years and a coupon of 5% per annum. Coupons are paid yearly in arrears on 15 August and the bond will be redeemed at 120% nominal. An investor purchased £20,000 nominal of this bond on 16 August 2008 for £22,000.

   Coupons and redemption proceeds were linked to the inflation index below with a 2-month time lag. In August 2009 the recorded values of the inflation index were:

Date	Index
Base	192.2
August 2005	194.1
June 2008	216.8
August 2008	217.8
September 2008	218.4
December 2008	212.9
February 2009	213.1
March 2009	211.3
June 2009	213.4

   (a) Assuming inflation of 5% per annum from June 2009, what will the redemption proceeds be?

   (b) Another investor bought £20,000 nominal of this bond on 16 February 2009. Assuming inflation of 5% per annum from 16 August 2009, what price did the investor pay if he achieved an effective rate of return of 10% per annum?

2) Denote the rate of return on a fund in year n by i_n. The rate of return in any given year is independent of the rates of return earned on the fund in all other years. It is assumed that 1 + i_n has a lognormal distribution with parameters μ and σ in any year n. The mean and variance of i_n are 0.08 and 0.04, respectively. Let S_n denote the accumulated value at the end of n years of 1 unit invested at time 0.

   (a) Determine the values of μ and σ

   (b) Determine the distribution of S_{10}

   (c) Calculate the probability that at time 10 the accumulated amount of an investment made at time 0 is less than 75% of the initial investment

Section 2

This section involves the application of force of interest and continuously paid cashflows. It also brings together the concept of net present value which is initially covered in Actuarial and Financial Maths A.

Examples

1) The force of interest is a function of time and at any time t, where time is measured in years, is given by

$$\delta(t) = \begin{cases} 0.02 \text{ for } 0 \leq t \leq 5 \\ 0.07 \text{ for } t \geq 5 \end{cases}$$

   (a) Calculate the accumulated value at time 10 of £100 invested at time 0.

   (b) Calculate the present value at time 0 of a continuous payment stream paid at the rate £500 per annum between times t = 8 and t = 10.

2) An investment project has a lifetime of 10 years. It incurs costs of £200000 at the start of first year. The net income from the project is received continuously from the start of the second year until the end of the tenth year. The initial rate of payment is £45000 per annum and this rate grows continuously at an effective rate of 4% per annum.

   (a) Calculate the net present value of the project at an effective rate of interest of 6% per annum.

   (b) Calculate the discounted payback period of the project at an effective rate of interest of 6% per annum.

Section 3

This section applies the concepts of duration, convexity and immunisation to solve exam questions. Forward contracts and the arbitrage concept are covered here.

Examples

1) An insurance company must make payments continuously on its annuity portfolio for the next 20 years. The continuous rate of payment is £1 million per annum. The government bond with the longest duration in which the insurance company can invest its funds is a zero coupon bond which is redeemed at par in 4 years' time. The current effective rate of interest is 5% per annum.

   (a) Calculate the duration of the insurance company's liabilities.

   (b) Without doing any further calculations, explain why the insurance company cannot immunise its liabilities by buying government bonds.

   (c) Show that for n > 0,

$$\int_0^n t^2 e^{-\delta t}\, dt = \frac{2(\bar{I}\bar{a})_{\overline{n|}} - n^2 v^n}{\delta}$$

   (d) The insurance company purchases two zero-coupon bonds issued by a corporation. One of the zero-coupon bonds pays £12.61 million in 5 years time and the other pays £7.68 million in 20 years time. Investigate whether the insurance company satisfies the necessary conditions to be immunised against small changes in the rate of interest.

2)

   (a) A forward contract with a delivery time of T is issued on a security. The security does not pay any dividends and has a current market price of S. The annualised risk-free force of interest is δ. Assuming no-arbitrage, show that the theoretical price of the forward contract is $K = Se^{\delta T}$

(b) An investor takes a short position in a forward contract on a non-dividend-paying stock, with a delivery time in 18 months. At the start of the contract, the stock price is £9.50 and the risk-free force of interest is 6% per annum. After one year, the stock price is £7.60 and the risk-free force of interest is 5.5% per annum.

Calculate the value to the investor of the forward contract one year after the start of the contract.

Section 4

This section covers forward rates and the term structure of interest rates.

Examples

1)

(a) Given a 2-year forward rate for lending and borrowing at time 1 of $f_{1,2} = 0.05$ and a 3-year forward rate at time 3 of $f_{3,3} = 0.07$, what would be the 5-year forward rate at time 1, $f_{1,5}$ under no-arbitrage conditions?

(b) In the same market as part(a), the price for a the price for a 6-year unit zero-coupon bond is $P_6 = 0.65$ What is the 1-year spot rate of interest, y_1, under no-arbitrage?

2)

(a) Explain what is meant by the “expectations theory” explanation for the shape of the yield curve.

(b) Explain how expectations theory can be modified by both “liquidity preference” and “market segmentation” theories.