Revision Session - Derivative Markets

Section 1

(1) • Long Position: 2 October Futures Contracts (on strawberry milk)

• Each contract: Delivery of 7500 pounds
Initial margin = \$3,000
Maintenance margin = \$2,250

• F_0^T = 80 cents per pound

(a) Margin Call when margin falls by \$750
↳ Long position
i.e. if the price falls by $\frac{\$750}{7500 \text{ pounds}} = \$0.10$ per pound
⇒ Margin Call if Price falls below 70 cents per pound

(b) If the Margin Call is not met, most often the trader's broker will close out some of his positions until it reaches the required margin. Any losses incurred are the trader's losses. Also, the broker might charge an amount as commission for the action

(c) \$1,000 to be withdrawn from the margin account:

Initial Margin = \$6,000 (2 contracts)

↳ \$1,000 increase for 15000 pounds
⇒ $\frac{\$1000}{15000} = \$\frac{1}{15}$ = 6.67 cents per pound

⇒ when Price rises to 86.67 cents per pound

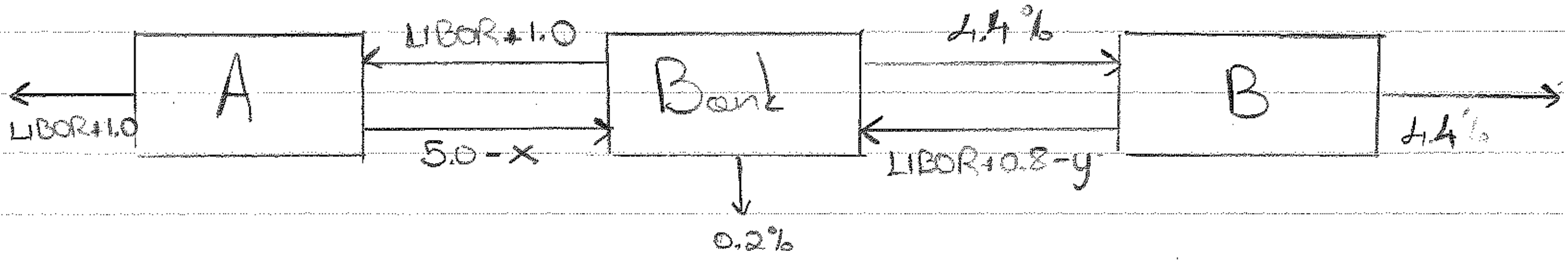
Section 2

(1) (a)

Company	Fixed	Floating
A	5.0%	LIBOR + 1.0%
B	4.4%	LIBOR + 0.8%

$\Rightarrow$ Total Benefit $= 0.6 - 0.2$
$= \underline{0.4\%}$

Commission of Bank : 0.2%



Bank Inflows = Bank Outflows

$\Rightarrow$ LIBOR + 1.0% + 4.4% + 0.2% = 5.0% − x + LIBOR + 0.8% − y

$\Rightarrow$ $x + y = 0.2\%$ $\Rightarrow$ $\boxed{x = y = 0.1\%}$

$\therefore$ A will be paying 4.9% fixed
B will be paying LIBOR + 0.7% floating

(b) Principal = £10 million 4.9% fixed $\leftrightarrow$ LIBOR
T = 9 months
Payments $\rightarrow$ semi-annually

i) Coupon = 10 million $\times \frac{0.049}{2}$ = £ 0.245 million (2 remaining)

$\Rightarrow B_{fix} = 0.245\, e^{-L_3\left(\frac{3}{12}\right)} + 10.245\, e^{-L_9\left(\frac{9}{12}\right)} = £\, 10.1325$ million

ii) $B_{fl} = (P + v)(1+R)^{-1} = \left(10 + 10 \times \frac{0.045}{2}\right)\left(1 + \frac{0.0444}{4}\right)^{-1} = £\, 10.1137$ million

Principal Variable payment in 3 months

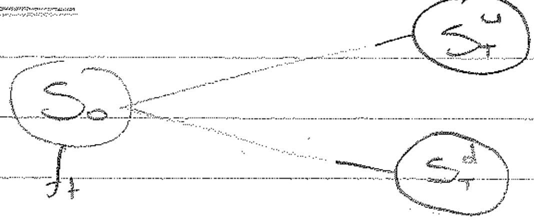
iii) $V_{fl} = B_{fl} - B_{fx} = -0.01875$ million $\approx$ \$ 18751

(c) Treasury Rates: The rate at which a government is borrowing in its own currency

LIBOR Rates: The rate at which large international banks will lend to each other

Section 3

(1)



$$B_t = B_0 e^{rt}$$

(a) Arbitrage free: $\tilde{S}_T^u > \tilde{S}_0 > \tilde{S}_T^d$

or $S_T^u > S_0 e^{rt} > S_T^d$

(b) i) Riskless Position: $V_T^u = V_T^d$

$$\Rightarrow \delta S_T^u + f_T^u = \delta S_T^d + f_T^d \Rightarrow \delta = \frac{f_T^d - f_T^u}{S_T^u - S_T^d}$$

ii) For a riskless portfolio, its value changes at the riskless rate

iii) Overall position: $(f_0 + \delta S_0)e^{rT} = f_T^u + \delta S_T^u = f_T^d + \delta S_T^d$

$$\Rightarrow f_0 e^{rT} = \delta S_T^u - \delta S_0 e^{rT} + f_T^u$$

$$\Rightarrow f_0 = \frac{f_T^d - f_T^u}{S_T^u - S_T^d} S_T^u e^{-rT} + f_T^u e^{-rT} - \frac{f_T^d - f_T^u}{S_T^u - S_T^d} S_T^d$$

$$f_0 = \left[\frac{S_T^u - S_0 e^{rT}}{S_T^u - S_T^d} f_T^d + \frac{S_0 e^{rT} - S_T^d}{S_T^u - S_T^d} f_T^u\right] e^{-rT}$$

iv) $f_0 = e^{-rT} E_Q(f_T)$ where $q^u = \frac{S_T^u - S_0 e^{rT}}{S_T^u - S_T^d}$

$q^d = \frac{S_0 e^{rT} - S_T^d}{S_T^u - S_T^d}$

Note: $q^u + q^d = 1$

↳ From no-arbitrage $0 < q^d, q^u < 1$

(c) i) <u>Options</u>: Convex payoffs $\rightarrow$ By Jensen's Inequality the derivative price increases as volatility increases

ii) <u>Butterfly Spreads</u>: Concave payoff $\rightarrow$ Derivative price decreases as volatility increases

iii) <u>Futures/Forwards</u>: Linear payoff $\rightarrow$ will not impact the derivative price

<u>Section 4</u>

(a) $\Omega = \{1,2,3,4\}$ and $C = \{\phi, \{1,4\}\}$

i) • Key subset of C is $\{2,3\}$

• Let $A = \{2,3\}$, $A^c = \{1,4\}$

$\Rightarrow$ Smallest σ-algebra: $\{\phi, A, A^c, \Omega\} = \{\phi, \{2,3\}, \{1,4\}, \Omega\}$

ii) Atoms: $A_1 = \{3\}$, $A_2 = \{2,4\}$, $A_3 = \{1\}$, $A_1 \cup A_2 \cup A_3 = \Omega$

$\Rightarrow$ Smallest σ-algebra: $\{\phi, A_1, A_2, A_3, A_1 \cup A_2, A_1 \cup A_3, A_2 \cup A_3, \Omega\}$
$= \{\phi, \{1\}, \{3\}, \{2,4\}, \{1,3\}, \{1,2,4\}, \{3,2,4\}, \Omega\}$

(b) Previsible $\rightarrow$ X_j is F_{j-1} measurable
Adapted $\rightarrow$ X_j is F_j measurable

(c) i) $\Omega = \{PPP, PPF, PFP, PFF, FPP, FPF, FFP, FFF\}$

ii) Before the game the σ-algebra is the trivial one $\Sigma_0 = \{\phi, \Omega\}$

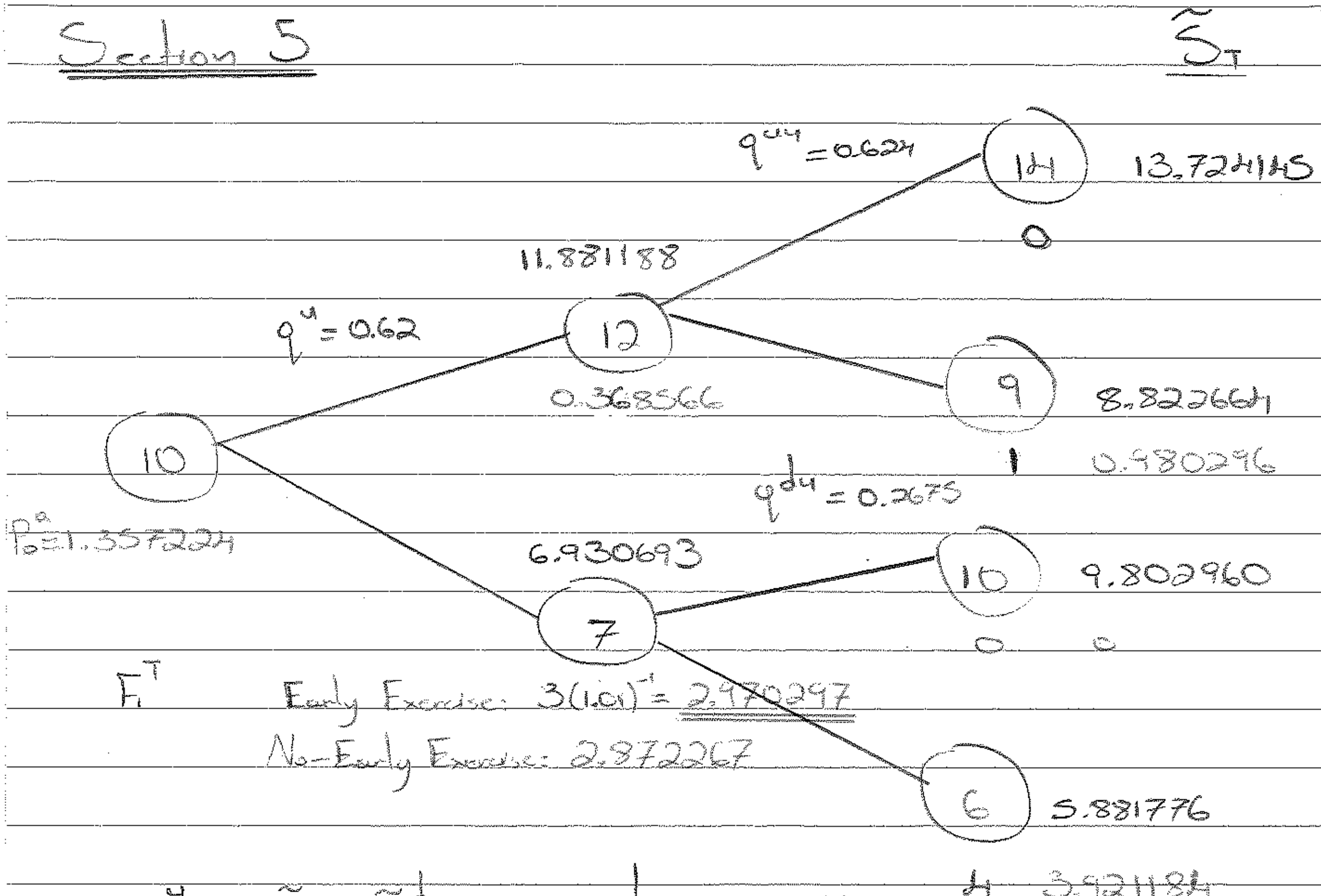
iii) $A = \{PPP, PPF, PFP, PFF\}$ $\Rightarrow$ $\Sigma_1 = \{\phi, A, A^c, \Omega\}$

iv) There are 16 elements to list down

Let $B_1 = \{PPP, PPF\}$, $B_2 = \{PFP, PFF\}$, $C_1 = \{FPP, FPF\}$, $C_2 = \{FFF, FFP\}$

$$\Rightarrow \Sigma_2 = \{\Phi, B_1, B_2, C_1, C_2, B_1 \cup B_2, B_1 \cup C_1, B_1 \cup C_2, B_2 \cup C_1, B_2 \cup C_2, C_1 \cup C_2, B_1 \cup B_2 \cup C_1, B_1 \cup B_2 \cup C_2, B_2 \cup C_1 \cup C_2, B_1 \cup C_1 \cup C_2, \Omega\}$$

Section 5



(a) $$q_j^u = \frac{\tilde{S}_j - \tilde{S}_{j+1}^d}{\tilde{S}_{j+1}^u - \tilde{S}_{j+1}^d}, \quad q_j^d = 1 - q_j^u$$

$$\Rightarrow q_1^u = \frac{10 - 6.930693}{11.881188 - 6.930693} = 0.6200$$

$$\Rightarrow q_2^{uu} = \frac{11.881188 - 8.822664}{13.724145 - 8.822664} = 0.6240$$

$$\Rightarrow q_2^{du} = \frac{6.930693 - 5.881776}{9.802960 - 5.881776} = 0.2675$$

(b) $K = 10$, $P_0^a = \underline{1.357224}$

(c) i) $\delta_1 = \dfrac{\tilde{f}_1^d - \tilde{f}_1^u}{\tilde{S}_1^u - \tilde{S}_1^d} = \dfrac{2.970297 - 0.368566}{11.881188 - 6.930693} = 0.525550$

Short 0.52555 assets

Also, $\rho_1 = -\delta_1 \tilde{S}_0 - \tilde{f}_0 = -0.52550(10) - 1.357224 = -6.612224$

Deposited in Bank

ii) If Asset Price drops to 7 at $t=1$:

$V_1 = 6.612224(1.01) - 0.525550(7) - \max(10-7, 0) = \underline{\underline{0}}$

<u>Position:</u> Short 0.52555 assets
Deposit 6.612224 in Bank
Short 1 Put with $K=10$